

Citation for published version:

Kong, W, Ma, K & Wu, Q 2018, 'Three Phase Power Imbalance Decomposition into Systematic Imbalance and Random Imbalance', *IEEE Transactions on Power Systems*, vol. 33, no. 3, pp. 3001-3012.
<https://doi.org/10.1109/TPWRS.2017.2751967>

DOI:

[10.1109/TPWRS.2017.2751967](https://doi.org/10.1109/TPWRS.2017.2751967)

Publication date:

2018

Document Version

Peer reviewed version

[Link to publication](#)

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Three Phase Power Imbalance Decomposition into Systematic Imbalance and Random Imbalance

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Abstract—Uneven load allocations and random load behaviors are two major causes for three-phase power imbalance. The former mainly cause systematic imbalance, which can be addressed by low-cost phase swapping; the latter contribute to random imbalance, which requires relatively costly demand-side managements. To reveal the maximum potential of phase swapping and the minimum need for demand-side managements, this paper first proposes a novel a priori judgment to classify any set of three-phase power series into one of four scenarios, depending on whether there is a definite maximum phase, a definite minimum phase, or both. Then, this paper proposes a new method to decompose three-phase power series into a systematic imbalance component and a random imbalance component as the closed-form solutions of quadratic optimization models that minimize random imbalance. A degree of power imbalance is calculated based on the systematic imbalance component to guide phase swapping. Case studies demonstrate that 72.8% of 782 low voltage substations have systematic imbalance components. The degree of power imbalance results reveal the maximum need for phase swapping and the random imbalance components reveal the minimum need for demand side management, if the three phases are to be fully rebalanced.

Index Terms— low voltage distribution network, power imbalance, random imbalance, systematic imbalance, three phase electric power

I. NOMENCLATURE

$DPIB(t)$	The degree of power imbalance at time point t
N	The total number of time points
$P_{\emptyset}(t)$ where $\emptyset \in \{a, b, c\}$	Phase \emptyset power at time point t
\bar{P}_{\emptyset} where $\emptyset \in \{a, b, c\}$	The average power of phase \emptyset over time
$P_{\emptyset s}(t)$ where $\emptyset \in \{a, b, c\}$	Phase \emptyset power of the systematic imbalance component at time point t

$x_r(t), y_r(t), z_r(t)$	The random imbalance component of the three-phase power (phases x, y, and z) at time point t
$x_s(t), y_s(t), z_s(t)$	The systematic imbalance component of three-phase power (phases x, y, and z) at time point t

II. INTRODUCTION

MORE than 70% of the UK's low voltage (LV) networks experience observable degrees of three-phase imbalance [1]. Such an imbalance leads to: 1) neutral wire energy losses up to hundreds of millions of British pounds each year in the UK's distribution networks [2], [3]; and 2) additional network investment cost amounting to billions of British pounds each year [4], [5]. Major causes for this issue are uneven load allocations across the three phases and random load behaviors [6], [7], [8].

Uneven load allocations cause systematic imbalance (SIB) where there is a definite maximum phase (a definite phase with the greatest power among the three phases), a definite minimum phase (a definite phase with the least power among the three phases), or both. SIB can be addressed by phase swapping [9], [10], [11], i.e., moving single-phase loads/laterals from one phase to another, which is a relatively cheap and mature technique.

Random load behaviors, on the other hand, are a major contributor to random imbalance (RIB) with neither a definite maximum phase nor a definite minimum phase. RIB requires demand-side managements [12], [13] to address, which incur relatively high implementation and operation costs (including the costs for per-phase monitoring, communication, and control systems) and a risk of non-delivery.

The motivation and objective of this paper is therefore to find a way to decompose any set of time series power data from three phases into a SIB component and a RIB component that reveal the maximum potential for phase swapping and the minimum need for demand-side managements, thus corresponding to the lowest cost to rebalance three-phase supply. This idea is

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analogous to the decomposition of physics experiment observational errors into systematic errors and random errors [14]: systematic errors result from the non-ideal mechanism (analogous to the non-ideal load allocation across three phases) of the experiment. It has a non-zero mean and is not reduced when observations are averaged [14]. Random errors, on the other hand, result from inherently unpredictable fluctuations [14], which are analogous to the random individual load fluctuations.

Three-phase power imbalance is the direct result of voltage/current imbalances [15]. The majority of publications studied the underlying power imbalance components, i.e., voltage/current imbalances: References [16], [17], [18] estimated voltage imbalance of medium-voltage (MV) distribution networks. Reference [19] assessed the sequence values of imbalanced voltages without phasor measurements. Reference [20] quantified current imbalance on short transmission lines. Reference [21] forecasted voltage imbalance on low voltage feeders with photovoltaic (PV) generation. Reference [22] converts three-phase imbalanced currents into two orthogonal AC currents with equal amplitudes. The above references implicitly decompose three-phase power imbalance into voltage and current imbalances.

A number of publications that focus on power imbalance are about reducing power imbalance [15], [23], [13], rather than on decomposing power imbalance into its underlying components.

The decomposition of three-phase imbalanced power series into a SIB component and a RIB component is a gap. The purpose for the decomposition is mentioned above. To bridge the gap, this paper makes the following contributions:

1) Propose a novel a priori judgment method to classify any set of three-phase power series into one of the following four scenarios: definite-max, definite-order, definite-min, and random imbalance scenarios (their definitions are given in Section III). The judgment method takes into account both the percentage of time when the definite phase occurs and the average power to ensure a robust judgment.

2) Propose a novel three-phase power decomposition method for all scenarios except the random imbalance one to decompose three-phase power series into a SIB component and a RIB component, which are the closed-form solution to a quadratic optimization problem that minimizes the RIB component.

3) Define the degree of power imbalance for each of the definite-max, definite-order, and definite-min scenarios based on the SIB component obtained from 2) and calculate the trend of the degree of power imbalance over time.

The SIB component, as a direct consequence of uneven load allocations, serves as the basis for calculating the degree of power imbalance, which provides a direct guidance for phase swapping; the RIB component, as a result of random individual load behaviors, indicates at least how much power on each phase has to be reduced by demand-side managements, if the three phases are to be fully rebalanced.

Therefore, the research outcome brings three values: 1) the decomposition helps distribution network operators (DNOs) to understand the potential (also the maximum need) of phase

swapping to address SIB and how much power on each phase has to be reduced by demand-side managements, if the three phases are to be fully rebalanced; 2) By calculating the degree of power imbalance based on the SIB component, the research also reveals the underlying trend of the SIB over time, reflecting the trend of uneven load allocations – this is particularly useful when increasing single-phase electric vehicles and heat pumps are connected to low voltage networks, causing the SIB to vary over time; 3) the degree of power imbalance also provides a guidance for phase swapping practices.

The remainder of the paper is organized as follows: Section III presents an overview of the methodology; Section IV presents a new a priori judgment method; Section V details the decomposition method; Section VI defines the degree of power imbalance; Section VII performs a case study; and Section VIII concludes the paper.

III. OVERVIEW OF METHODOLOGY

The proposed methodology requires three-phase power series as an input only. Therefore, as a mathematical method, it is applicable to where: 1) there is monitoring of three-phase power (or three-phase voltages and currents which can be used to derive power); and 2) there is three-phase power imbalance. In reality, the methodology is highly suitable for monitored low voltage distribution networks in the UK and the rest of Europe and monitored medium voltage distribution networks in the US, where three-phase power imbalance is obvious.

Fig. 1 shows an overview of the methodology.

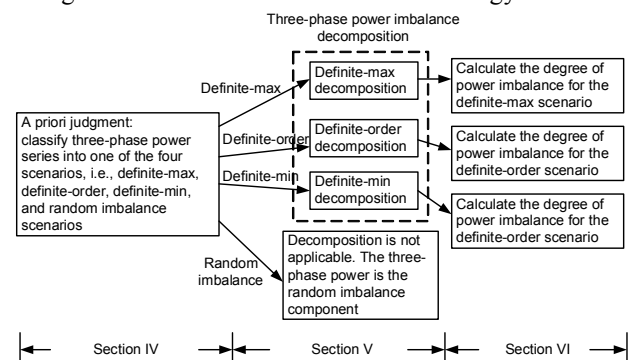


Fig. 1. An overview of methodology.

Each phase has a time series of power (called a power series) monitored at the LV (415V) substation side. The following definitions are used throughout the paper:

1) Three-phase power series: a set of three time series of power data monitored and collected from three phases. The data are normally measured from distribution substations at an interval of T_{int} , e.g., $T_{int} = 10$ min.

2) Definite-max phase: a definite phase with the greatest power among three phases.

3) Definite-max scenario: the scenario where there is a definite-max phase for the majority of time.

4) Definite-min phase: a definite phase with the least power among three phases.

5) Definite-min scenario: the scenario where there is a definite-min phase for the majority of time.

6) Definite order: the existence of both a definite-max phase and a definite-min phase, e.g., ‘phase a > phase b > phase c’.

7) Definite-order scenario: the scenario where there are both definite-max and definite-min phases for the majority of time.

8) Random imbalance scenario: the scenario where there is neither a definite-max phase nor a definite-min phase.

9) SIB component: a set of three-phase power series with a definite-max phase, a definite-min phase, or a definite order.

10) RIB component: a set of three power series with neither a definite-max phase nor a definite-min phase.

IV. A PRIORI JUDGMENT

This section presents a new a priori judgment method to classify any set of three-phase power series into one of the four scenarios (definite-max, definite-order, definite-min, and random imbalance scenarios). The judgment considers both the percentage of time when a definite phase/order occurs and the average power. The rationale for this is to ensure robustness: the definite phase/order, if exist, should not only occur for the majority of time but also have the average power showing the same trend. The judgment method consists of three steps:

Step 1): The percentage of time judgment

In principle, Step 1) judgment indicates that:

1) If for the majority of time, phase a is the definite-max phase and phase c is the definite-min phase, then this is a definite-order scenario with a definite three-phase order: phase a > phase b > phase c.

2) If condition 1) is *not* met, and phase a has the greatest power among the three phases for the majority of time which is no less than the time when any phase has the least power among the three phases, then this is a definite-max scenario where phase a is the definite-max phase.

3) If condition 1) is *not* met, and phase c has the least power among the three phases for the majority of time which is more than the time when any phase has the greatest power among the three phases, then this is a definite-min scenario where phase c is the definite-min phase.

4) Any scenario that does not meet conditions 1) – 3) is a random imbalance scenario with neither a definite-max phase nor a definite-min phase.

The percentage of time when phases a is the definite-max phase and phase c is the definite-min phase is given by,

$$C_{ac} = \frac{\sum_{t=1}^N \alpha_a(t) \beta_c(t)}{N} \times 100\% \quad (1)$$

where N is defined in Section I. $\alpha_a(t)$ is a binary value:

$$\alpha_a(t) = \begin{cases} 1 & \text{when } P_a(t) > (1 + \delta_1)P_b(t) \\ & \text{and } P_a(t) > (1 + \delta_1)P_c(t) \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where $P_\phi(t)$ is defined in Section I. δ_1 is a threshold to distinguish any two power values, e.g., $\delta_1 = 5\%$. Such a threshold accounts for measurement errors, which arise from monitoring devices, the communication system, and other factors. This value is chosen according to network operator’s experience. If the difference between two power values is below this threshold, then the difference is immersed in the

measurement error and is not regarded as a credible difference. In this paper, $\delta_1 = 5\%$ by default.

$\beta_c(t)$ is also a binary value:

$$\beta_c(t) = \begin{cases} 1 & \text{when } P_c(t) < (1 - \delta_1)P_a(t) \\ & \text{and } P_c(t) < (1 - \delta_1)P_b(t) \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

The percentage of time when each phase a has the greatest power is given by,

$$A_a = \frac{\sum_{t=1}^N \alpha_a(t)}{N} \times 100\% \quad (4)$$

Where $\alpha_a(t)$ is given by (2).

Similarly, the percentage of time when each phase c has the least power is given by,

$$B_c = \frac{\sum_{t=1}^N \beta_c(t)}{N} \times 100\% \quad (5)$$

where $\beta_c(t)$ is given by (3). Based on the results from (1), (4), and (5), Step 1) judgment is listed in Table I.

TABLE I
STEP 1) JUDGMENT

Case ID	Condition	Step 1) judgment
1	If \exists phase a, such that $C_{\phi_1\phi_2} < 50\%$ and $A_a \geq 50\%$ and $A_a \geq B_\phi$, where $\phi_1, \phi_2, \phi \in \{a, b, c\}$	Definite-max scenario: phase a is the definite-max phase
2	If \exists phase c, such that $C_{\phi_1\phi_2} < 50\%$ and $B_c \geq 50\%$ and $B_c \geq A_\phi$, where $\phi_1, \phi_2, \phi \in \{a, b, c\}$	Definite-min scenario: phase c is the definite-min phase
3	If \exists phases a and c, such that $C_{ac} \geq 50\%$	Definite-order scenario: phase a > phase b > phase c
4	Other	Random imbalance scenario

The 50% threshold of time is consistent with the criteria detailed at the beginning of this section, where the term “majority” means a 50% threshold by default.

It should be noted that Step 1) produces preliminary judgment results which are not necessarily the final ones.

Step 2): Calculation of the average power

The average power of each phase ϕ is given by,

$$\bar{P}_\phi = \frac{\sum_{t=1}^N P_\phi(t)}{N} \quad \text{where } \phi \in \{a, b, c\} \quad (6)$$

Where all variables are defined in Section I.

The resultant set of the three average power $\{\bar{P}_a, \bar{P}_b, \bar{P}_c\}$ will be used for judgment in Step 3).

Step 3): Combined judgment

Table II lists the final judgments of the scenarios (in the right column of Table II) as the combinations of the judgments from Step 1) and Step 2). It should be noted that the logic is ‘and’ between the conditions in the first (Step 1) and second (Step 2) columns.

TABLE II
FINAL JUDGMENTS

Step 1)	Step 2) condition	Combined judgment
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case ID		
1	If $\bar{P}_a = \max\{\bar{P}_a, (1 + \delta_1)\bar{P}_b, (1 + \delta_1)\bar{P}_c\}$	Definite-max scenario: phase a is the definite-max phase
	Otherwise	Random imbalance scenario
2	If $\bar{P}_c = \min\{\bar{P}_c, (1 - \delta_1)\bar{P}_a, (1 - \delta_1)\bar{P}_b\}$	Definite-min scenario: phase c is the definite-min phase
	Otherwise	Random imbalance scenario
3	If $\bar{P}_a > (1 + \delta_1)\bar{P}_b$ and $\bar{P}_b > (1 + \delta_1)\bar{P}_c$	Definite-order scenario: phase a > phase b > phase c
	If $\bar{P}_c = \min\{(1 - \delta_1)\bar{P}_a, (1 - \delta_1)\bar{P}_b, \bar{P}_c\}$	Definite-min scenario: phase c is the definite-min phase
	If $\bar{P}_a = \max\{\bar{P}_a, (1 + \delta_1)\bar{P}_b, (1 + \delta_1)\bar{P}_c\}$	Definite-max scenario: phase a is the definite-max phase
	Otherwise	Random imbalance scenario
4	Any	Random imbalance scenario

δ_1 is the same threshold as appeared in (2). The left column, Step 1) case ID, corresponds to the case ID in Table I.

V. POWER IMBALANCE DECOMPOSITION

According to the priori judgment in Section III, three-phase power series are classified into four scenarios, i.e., definite-max, definite-order, definite-min, and random imbalance scenarios. For the first three scenarios, this section presents three decomposition methods: definite-max decomposition, definite-order decomposition, and definite-min decomposition. Each decomposition corresponds to a quadratic optimization problem that minimizes the RIB component. The fourth scenario, i.e., the random imbalance scenario, cannot be decomposed.

The quadratic optimization problems have closed-form solutions. Therefore, the decompositions are achieved by directly applying the closed-form solutions without the need for iterations to solve the optimization problems. This significantly simplifies the decomposition process and ensures that the decomposition only has linear complexity (linear to the length of the three-phase power series).

The resultant SIB can be addressed by phase swapping, which is a low frequency, relatively long-lasting, mature solution. However, phase swapping is not suitable for resolving RIB which does not have a particular phase order. The RIB requires solutions such as demand-side managements, which incur higher monitoring, communication, and control costs as well as a risk of non-delivery. Therefore, this justifies the principle of the three decompositions to minimize the RIB component, thus revealing the maximum potential of phase swapping (i.e., the maximum possible reduction in phase imbalance from phase swapping) as well as the minimum need for demand-side managements.

The purposes of the decomposition are twofold: i) to serve as the basis to calculate the degree of power imbalance (in Section VI), which not only reveals the potential of phase swapping to address the SIB but also serves as a guidance for phase swapping; ii) to understand how much power on each phase needs to be reduced by demand-side managements, if the three phases are to be fully rebalanced.

A. Definite-Max Decomposition

The definite-max decomposition decomposes imbalanced three-phase power series into: 1) a SIB component with a definite-max phase; and 2) a RIB component without the definite-max phase.

The definite-max decomposition applies to the definite-max scenario, in which phase a is defined as the definite-max phase. The definite-max decomposition is the solution to the following quadratic optimization problem:

$$\begin{aligned}
 & \min \sum_{t=1}^N x_r^2(t) + y_r^2(t) + z_r^2(t) \\
 & \text{subject to } P_a(t) = x_r(t) + x_s(t) \\
 & \quad P_b(t) = y_r(t) + y_s(t) \\
 & \quad P_c(t) = z_r(t) + z_s(t) \\
 & \quad x_s(t) \geq y_s(t) \\
 & \quad x_s(t) \geq z_s(t) \\
 & \quad x_s(t), y_s(t), z_s(t) \geq 0 \\
 & \quad x_r(t), y_r(t), z_r(t) \geq 0
 \end{aligned} \tag{7}$$

Where all variables are defined in Section I.

The quadratic optimization problem given by (7) aims to minimize the RIB component that requires demand-side management to address. This is justified because demand-side management, which addresses RIB, incurs relatively high implementation and operation costs and a risk of non-delivery. On the other hand, phase swapping, which addresses systematic imbalance, is a relatively economic and mature technique. By minimizing RIB (hence maximizing SIB), the quadratic optimization model aims to reveal the minimum need for demand-side management and the maximum need for phase swapping, thus corresponding to the lowest cost. The same justification applies to the optimization problems for the definite-order and definite-min scenarios.

The original problem of (7) minimizing the summation of a time series is transformed into a total of N optimizations, each for a time point t . In this way, the summation is removed and the objective function of (7) becomes:

$$\forall t \in [1, N] \quad \min x_r^2(t) + y_r^2(t) + z_r^2(t) \tag{8}$$

A closed-form solution exists for the optimization problem in (8). The solution includes both the SIB and RIB components, assuming that phase a is the definite-max phase:

$$\text{SIB} = [P_a(t), P_{2s}(t), P_{3s}(t)]^T \tag{9}$$

where

$$\begin{aligned}
 P_{2s}(t) &= \min\{P_a(t), P_b(t)\}; \\
 P_{3s}(t) &= \min\{P_a(t), P_c(t)\}.
 \end{aligned}$$

Because

$$\text{SIB} + \text{RIB} = [P_a(t), P_b(t), P_c(t)]^T \tag{10}$$

The RIB component is given by

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$$\text{RIB} = [0, P_{2r}(t), P_{3r}(t)]^T \quad (11)$$

where

$$\begin{aligned} P_{2r}(t) &= \max\{0, P_b(t) - P_a(t)\}; \\ P_{3r}(t) &= \max\{0, P_c(t) - P_a(t)\}. \end{aligned}$$

B. Definite-Order Decomposition

The definite-order decomposition decomposes imbalanced three-phase power series into: 1) a SIB component with a definite-order; and 2) a RIB component without the definite-order.

Suppose that the phase order is 'a > b > c'. The quadratic optimization model is the same as given by (7) except that the first two inequality constraints are replaced by

$$\begin{aligned} x_s(t) &\geq y_s(t) \\ y_s(t) &\geq z_s(t) \end{aligned} \quad (12)$$

The definite-order decomposition is the closed-form solution to the optimization model. Assuming that the order of the three phases is 'a > b > c', the SIB component is given by,

$$\text{SIB} = [P_a(t), P_{2s}(t), P_{3s}(t)]^T \quad (13)$$

where

$$\begin{aligned} P_{2s}(t) &= \min\{P_a(t), P_b(t)\}, \\ P_{3s}(t) &= \min\{P_a(t), P_b(t), P_c(t)\}. \end{aligned}$$

Equation (10) still holds. The RIB component is given by,

$$\text{RIB} = [0, P_{2r}(t), P_{3r}(t)]^T \quad (14)$$

where

$$\begin{aligned} P_{2r}(t) &= \max\{0, P_b(t) - P_a(t)\}; \\ P_{3r}(t) &= \max\{0, P_c(t) - P_a(t), P_c(t) - P_b(t)\}. \end{aligned}$$

The definite-order scenario provides more information than the definite-max and definite-min scenarios, because its SIB component gives a definite three-phase order with both definite-max and definite-min phases, whereas the SIB components of the latter two scenarios give only the definite-max or the definite-min phase. On the other hand, the definite-order scenario is more restrictive than the latter two because it requires that a definite three-phase order exists.

C. Definite-Min Decomposition

The definite-min decomposition decomposes imbalanced three-phase power series into: 1) a SIB component with a definite-min phase; and 2) a RIB component without the definite-min phase.

Suppose that the definite-min phase is phase c. The quadratic optimization model is the same as given by (7) except that the first two inequality constraints are replaced by

$$\begin{aligned} x_s(t) &\geq z_s(t) \\ y_s(t) &\geq z_s(t) \end{aligned} \quad (15)$$

The definite-min decomposition is the closed-form solution to the optimization problems. Assuming that phase c is the definite-min phase, the SIB component is given by,

$$\text{SIB} = [P_a(t), P_b(t), P_{3s}(t)]^T \quad (16)$$

where $P_{3s}(t) = \min\{P_a(t), P_b(t), P_c(t)\}$.

Equation (10) still holds. The RIB component is given by,

$$\text{RIB} = [0, 0, P_{3r}(t)]^T \quad (17)$$

where $P_{3r}(t) = \max\{0, P_c(t) - P_a(t), P_c(t) - P_b(t)\}$.

For the definite-max, definite-order, and definite-min scenarios, the SIB component is the basis for calculating the degree of power imbalance, which provides a direct guidance for phase swapping as explained in Section VI. The $\text{RIB} = [P_{1r}(t), P_{2r}(t), P_{3r}(t)]^T$ has a clear meaning: for phases a, b, and c, at least $P_{1r}(t)$, $P_{2r}(t)$, and $P_{3r}(t)$ of loads require demand-side managements for phase rebalancing, respectively.

VI. DEGREE OF POWER IMBALANCE

This section presents the definitions for the degree of power imbalance for the definite-max, definite-order, and definite-min scenarios. For all three scenarios, the degree of power imbalance is defined as the deviation of the definite-max/definite-min phase from the average, based on the SIB component. The definition of the degree of power imbalance is to not only reveal the trend of SIB over time but also guide phase swapping (as explained later in this section). Assume that phase a is the definite-max phase for the definite-max scenario; phase c is the definite-min phase for the definite-min scenario; and the phase order is 'a > b > c' for the definite-order scenario. The mathematical definition for the degree of power imbalance for each scenario is given by,

$$\text{Definite-max: } DPIB(t) = \frac{P_{as}(t) - \frac{\sum_{\theta \in \{a,b,c\}} P_{\theta s}(t)}{3}}{\frac{\sum_{\theta \in \{a,b,c\}} P_{\theta s}(t)}{3}} \times 100\% \quad (18)$$

$$\text{Definite-min: } DPIB(t) = \frac{\frac{\sum_{\theta \in \{a,b,c\}} P_{\theta s}(t)}{3} - P_{cs}(t)}{\frac{\sum_{\theta \in \{a,b,c\}} P_{\theta s}(t)}{3}} \times 100\% \quad (19)$$

$$\text{Definite-order: } DPIB(t) = [D_1 \ D_2] = \left[\frac{P_{as}(t) - \frac{\sum_{\theta \in \{a,b,c\}} P_{\theta s}(t)}{3}}{\frac{\sum_{\theta \in \{a,b,c\}} P_{\theta s}(t)}{3}}, \frac{\frac{\sum_{\theta \in \{a,b,c\}} P_{\theta s}(t)}{3} - P_{cs}(t)}{\frac{\sum_{\theta \in \{a,b,c\}} P_{\theta s}(t)}{3}} \right] \quad (20)$$

Where all variables are defined in Section I. $\text{SIB} = [P_{as}(t), P_{bs}(t), P_{cs}(t)]^T$ as given by (9), (13), and (16). It should be noted that for the definite-max or definite-min scenarios, the degree of power imbalance is a single value; but for the definite-order scenario, the degree of power imbalance is a vector of two values.

The average three-phase power of the SIB component is given by,

$$\bar{P}_s(t) = \frac{\sum_{\theta \in \{a,b,c\}} P_{\theta s}(t)}{3} \quad (21)$$

where $P_{\theta s}(t)$ is defined in Section I.

The degree of power imbalance is a time series. It brings three values by: 1) revealing the trend of the SIB over time, i.e., the trend of uneven load allocations – this is particularly useful when increasing single-phase electric vehicles are connected to the network; 2) showing the potential of phase swapping to address SIB; iii) and providing a direct guidance for phase swapping:

i) For the definite-max scenario, the degree of power imbalance suggests the move of loads totaling $3\bar{P}_s(t)DPIB(t)$ from the definite-max phase to the other two phases, where $\bar{P}_s(t)$ is given by (21). ii) For the definite-order scenario, the degree of power imbalance suggests the move of loads totaling $3\bar{P}_s(t)D_1$ away from the definite-max phase and the move of

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$3\bar{P}_s(t)D_2$ to the definite-min phase, where D_1 and D_2 are defined in (20). iii) For the definite-min scenario, the degree of power imbalance suggests the move of loads totaling $3\bar{P}_s(t)DP_{IB}(t)$ to the definite-min phase from the other two phases.

VII. NUMERICAL RESULTS

The input data are three-phase power series for 782 low voltage substations derived from the three-phase voltages and currents monitored at the secondary side of 11kV/415V transformers throughout Western Power Distribution (a UK DNO)'s business area [24]. Therefore, the three-phase power series are the power injected from 11kV networks to 415V networks. The data cover a good mix of geographical characteristics and customer types [24]. Four representative substations are selected to demonstrate the methodology. Matlab is used for the simulation.

A. Definite-Max Scenario

Substation No. 536,753 is selected to represent the definite-max scenario. The study period is one year, covering five seasons (spring, summer, high summer, autumn, and winter) and different day types (weekday and weekend). Because the original three-phase power series and the SIB component have more than 50,000 time points (one sample every 10 minutes for a year) on the X axis, they are presented in the form of probability density distributions for clarity. This also applies to the definite-order and definite-min scenarios. The probability density functions of the three-phase power series are presented in Fig. 2.

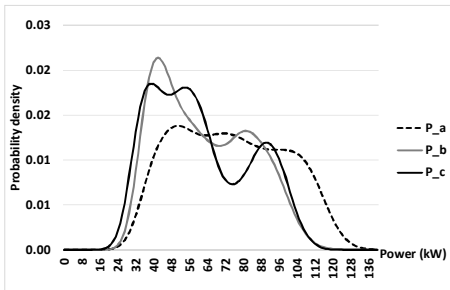


Fig. 2. The probability density functions of the three-phase power series over a year for definite-max scenario

The priori judgment process is presented in Table III.

TABLE III

A PRIORI JUDGMENT FOR THE DEFINITE-MAX SCENARIO

Sub No.	Variables	Phase a	Phase b	Phase c
536,753	A_θ	71.47%	7.15%	2.67%
	B_θ	1.66%	26.18%	37.88%
	\bar{P}_θ (kW)	74.25	62.63	61.03

A_θ , B_θ , and \bar{P}_θ are given by (4), (5), and (6), respectively.

Phase a is the definite-max phase. Although phase c has the least power among the three phases for the majority of time (as shown in the second row of Table III), its average power is approximately the same as that of phase b (their difference is lower than the threshold δ_1 as defined in (2)). Therefore, phase

c is not judged as the definite-min phase and only the definite-max phase exists in this case.

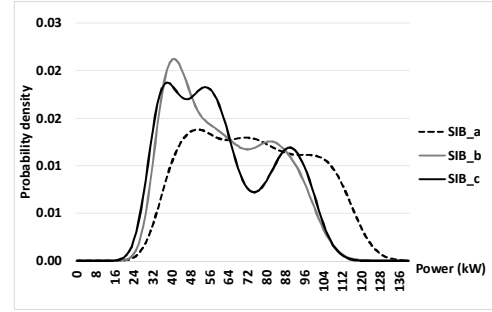


Fig. 3. The probability density functions of the SIB component over a year for definite-max scenario

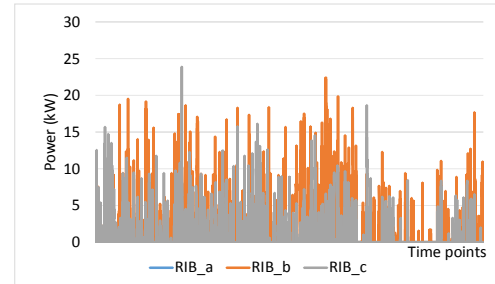


Fig. 4. The RIB component over a year for definite-max scenario

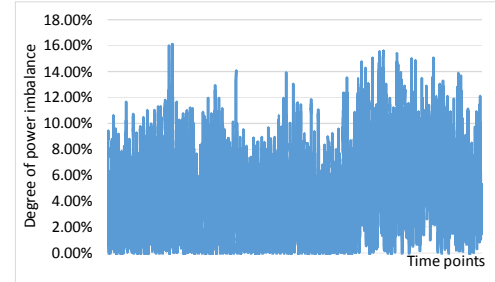


Fig. 5. The degree of power imbalance over a year for definite-max scenario

Similar as Fig. 2, the SIB component is also presented in the form of probability density functions in Fig. 3. It can be seen that the probability density functions in Fig. 3 look similar to those in Fig. 2 with only slight differences. In the SIB component, phase a has the greatest power among the three phases for 100% of the time – this is consistent with the definition of the SIB for the definite-max scenario, reflecting that too much load is allocated to phase a.

The RIB component presented in Fig. 4 shows the anomalies of when either phase b or c overtakes phase a to become the maximum phase – this occurs for 18.40% of the time, reflecting the random load fluctuations on phases b and c.

For the whole year, the degree of power imbalance results in Fig. 5 provide a guidance for phase swapping: 1) on average, up to 8.61kW of loads can be moved from phase a to the other two phases; 2) at 15:50 on the 80th day in the year (21st March), a maximum of 34.88kW of loads can be moved from phase a to the other two phases; 3) for 4.20% of time, no load needs to be moved from phase a to the other two phases (the degree of power imbalance is zero during this period). If phase swapping is performed to move loads away from phase a, then the loads on the other two phases need to be reduced for phase rebalancing during this minority period.

B. Definite-Order Scenario

Substation No. 512,457 is selected to represent the definite-order scenario. The probability density functions of the three-phase power series are presented in Fig. 6.

TABLE IV

A PRIORI JUDGMENT FOR THE DEFINITE-ORDER SCENARIO

Sub No.	Variables	Phase a	Phase b	Phase c
512,457	A_ϕ	83.35%	5.88%	0
	B_ϕ	0	1.10%	96.11%
	\bar{P}_ϕ (kW)	45.36	36.89	25.26

In this case, $C_{ac} = 79.86\%$. The average power also demonstrates the order of ‘a > b > c’. Therefore, phases a and c are the definite-max and definite-min phases, respectively.

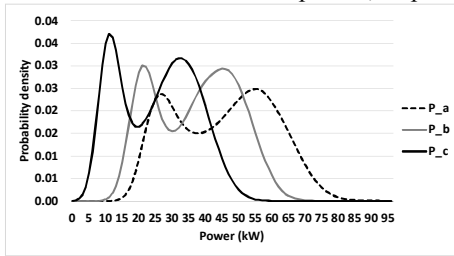


Fig. 6. The probability density functions of the three-phase power series over a year for definite-order scenario

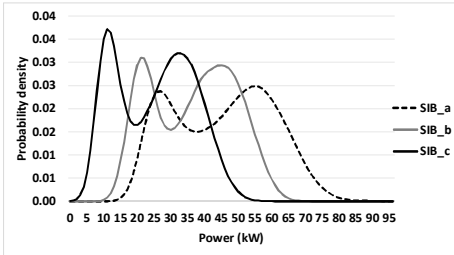


Fig. 7. The probability density functions of the SIB component over a year for definite-order scenario

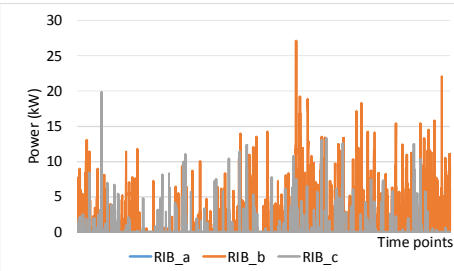


Fig. 8. The RIB component over a year for definite-order scenario

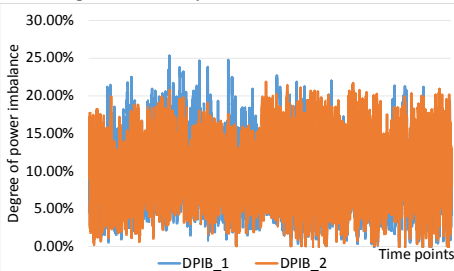


Fig. 9. The degree of power imbalance over a year for definite-order scenario

For the whole year, the probability density functions of the SIB component is presented in Fig. 7. According to the SIB

component, phase a has the greatest power among the three phases for 100% of the time and that phase c has the least power for 100% of the time – this is consistent with the definition of the SIB for the definite-order scenario, reflecting the existence of excessive loads on phase a and insufficient loads on phase c.

The RIB component presented in Fig. 8 shows the anomalies when the phase order is not ‘a > b > c’ – this occurs for 12.27% of the time, reflecting the random load fluctuations on each phase.

The degree of power imbalance results in Fig. 9 provide a guidance for phase swapping: 1) on average, up to 9.66kW of loads can be moved away from phase a and up to 10.50kW of loads can be moved to phase c; 2) at 22:30 on the 32nd day in the year (1st February), a maximum of 28.65kW of loads can be moved from phase a to the other two phases: 9.89kW to phase b and 18.76kW to phase c.

C. Definite-Min Scenario

Substation No. 521,071 is selected to represent the definite-min scenario. The three-phase power series are presented in Fig. 10. The priori judgment process is presented in Table V.

TABLE V

A PRIORI JUDGMENT FOR THE DEFINITE-MIN SCENARIO

Sub No.	Variables	Phase a	Phase b	Phase c
521,071	A_ϕ	0	67.62%	13.00%
	B_ϕ	69.26%	0	15.02%
	\bar{P}_ϕ (kW)	46.92	60.95	54.86

Phase a is the definite-min phase. Although phase b has a power greater than the other two phases by more than 5% for 67.62% of the time, the order of ‘b > c > a’ only occurs for 37.79% (< 50%) of the time – it does not meet the criteria for the definite-order scenario. Therefore, only a definite-min phase exists.

For a whole year, Fig. 10 – 13 depict the probability density functions of the three-phase power series, the probability density functions of the SIB component, the time series of the RIB component, and the degree of power imbalance, respectively.

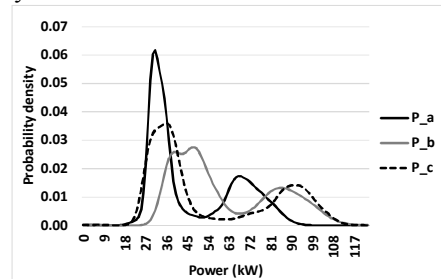


Fig. 10. The probability density functions of the three-phase power series over a year for definite-min scenario

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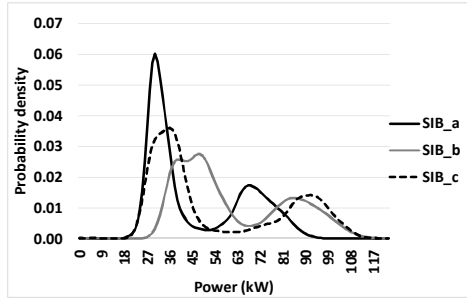


Fig. 11. The probability density functions of the SIB component over a year for definite-min scenario

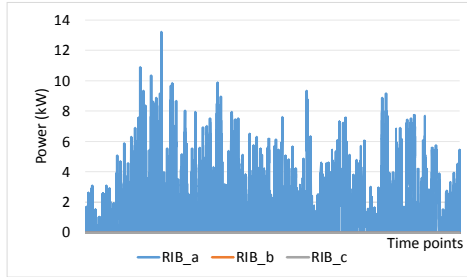


Fig. 12. The RIB component over a year for definite-min scenario

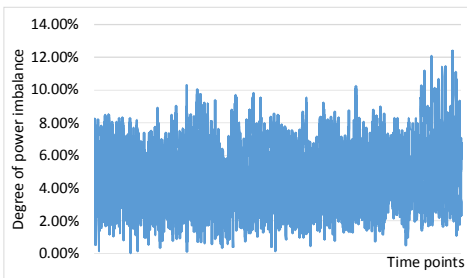


Fig. 13. The degree of power imbalance over a year for definite-min scenario

The SIB component presented in Fig. 11 shows that phase a has the least power among the three phases for 100% of the time – this is consistent with the definition of the SIB for the definite-min scenario, reflecting that insufficient loads are allocated to phase a.

The RIB component presented in Fig. 12 shows that the abnormal cases when phase a is not the minimum phase occur for 21.58% of the time, reflecting the random load fluctuations that breach the majority rule.

The degree of power imbalance results in Fig. 13 provide a guidance for phase swapping: 1) on average, up to 7.75kW of loads can be moved from phases b and c to phase a; 2) At the 14192nd time point (at 13:20, 8th April), a maximum of 21.48kW of loads can be moved from phases b and c to phase a.

D. Random Imbalance Scenario

Substation No. 521,064 is selected to represent the random imbalance scenario. The a priori judgment process is presented in Table VI.

TABLE VI
A PRIORI JUDGMENT FOR THE RANDOM IMBALANCE SCENARIO

Sub No.	Variables	Phase a	Phase b	Phase c
521,064	A_\emptyset	30.10%	21.03%	26.26%
	B_\emptyset	25.39%	18.21%	31.99%

\bar{P}_\emptyset (kW)	46.43	45.34	43.52
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Although phase c has the least power among the three phases for half of the time, its average power is not lower than that of phase b by more than the threshold δ_1 . Therefore, phase c is not judged as the definite-min phase. The three-phase power series then belong to the RIB scenario which cannot be decomposed into SIB and RIB. In this case, the three-phase power series are the RIB component. The probability density functions of the three-phase power series are presented in Fig. 14.

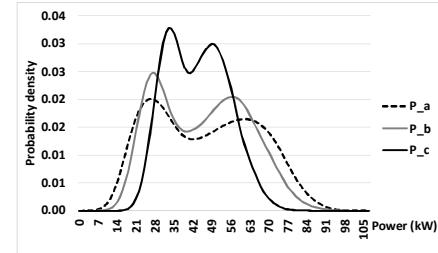


Fig. 14. The probability density functions of the three-phase power series over a year for random imbalance scenario

E. Impact of Majority Threshold on Decomposition

According to Table I, the majority threshold is the key for the a priori judgment. When the majority threshold is set as 50% (by default), 99.2% of the definite-max cases exhibit this feature: the average power of their definite-max phase is greater than those of the other two phases by more than 5% – this indicates that the majority threshold of 50% is reasonable because the order of the average power is consistent with the percentage of time when the definite-max occurs; similarly, 96.5% of the definite-order cases demonstrate the consistency between the order of the average power and the percentage of time when the order occurs; 97.8% of the definite-min cases demonstrate the consistency between the order of the average power and the percentage of time when the definite-min phase occurs. Therefore, the majority threshold of 50% is judged to be reasonable.

If, for example, the majority threshold is set as 60%, out of 782 substations, 14.07% (110) that were classified as definite-max, definite-order, and definite-min scenarios under the threshold of 50% are now classified as the random imbalance scenario under the new threshold of 60%. The threshold of 60% is not reasonable because those 14.07% (110) of substations actually have a definite-max phase, a definite-min phase, or both in terms of the average power, indicating the existence of systematic imbalance and the potential for phase swapping.

Table VII presents the a priori judgment results (i.e., the number of substations belonging to each scenario) under different majority thresholds.

TABLE VII
A PRIORI JUDGMENT RESULTS UNDER DIFFERENT MAJORITY THRESHOLDS

Majority threshold	Numbers of definite-max, definite-order, and definite-min cases, respectively
50%	235, 164, 170, 213
55%	220, 131, 169, 262

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60%	205, 101, 153, 323
65%	191, 77, 144, 370

Table VII shows that, with the increase of the majority threshold, the numbers of definite-max, definite-order, and definite-min cases all decrease but the number of random imbalance cases increases.

The impact of the majority threshold on three-phase power decomposition is derived from Table VII: each time the majority threshold increases by 5%, approximately 6.4% of the ‘decomposable’ cases (i.e., cases that can be decomposed into systematic imbalance and random imbalance) becomes ‘non-decomposable’ (i.e., belonging to the random imbalance scenario which cannot be decomposed).

However, as mentioned above, the increase of the majority threshold to over 50% masks the existence of systematic imbalance and the potential for phase swapping; the majority threshold of 50% is found to be reasonable.

F. Impact of Measurement Error Threshold on Decomposition

In Equations (2) and (3), there is a threshold δ_1 that accounts for measurement errors. How this threshold affects the a priori judgment results and consequently the decomposition is investigated in this section. Fig. 15 depicts the numbers of substations belonging to the four scenarios under different measurement error thresholds.

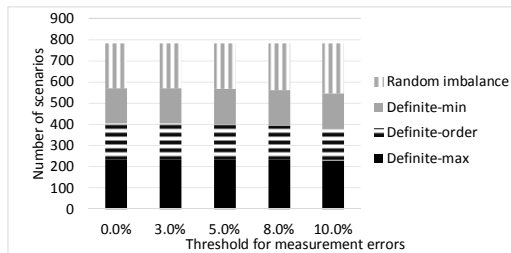


Fig. 15. The impact of measurement error threshold on judgment results

Fig. 15 shows that: 1) with the increase of the measurement error threshold from 0 to 10%, the number of random imbalance cases increases from 211 to 237, i.e., 26 more cases become ‘non-decomposable’ under the threshold of 10% as compared to that under the threshold of 0; 2) when the measurement error threshold is below 5%, the threshold has negligible impact on the a priori judgment results.

G. Validation by Phase Swapping

In this section, preliminary phase swapping is performed under the guidance of the degree of power imbalance to validate the methodology. Take Substation No. 536,753 (belonging to the definite-max scenario) as an example. Before phase swapping, its three-phase power series, the SIB component, the RIB component, and the degree of power imbalance are presented in Fig. 2 – 5, respectively.

The degree of power imbalance results suggest that the distribution network operator move an average load of 8.61kW from phase a to the other two phases. Therefore, a preliminary phase swapping strategy is to move 10 single-phase domestic

customers from phase a to phases b and c (5 customers to phase b and 5 customers to phase c). Suppose that the total load of these 10 customers follows a normal distribution with an average value of 8kW and a standard deviation of 3kW.

After phase swapping, the three-phase power series then belongs to the random imbalance scenario (the systematic component is zero). The RIB component equals the three-phase power series, the probability density functions of which are presented in Fig. 16.

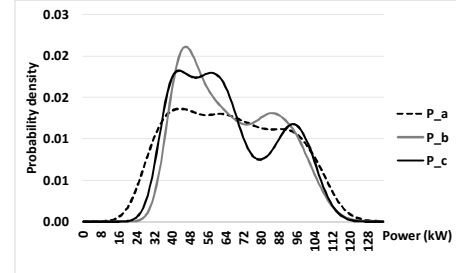


Fig. 16. Three-phase power series after phase swapping

The phase swapping eliminates systematic imbalance: after phase swapping, there is no phase that exhibits the greatest power among the three phases for more than 50% of the time; neither is there any phase that exhibits the least power among the three phases for more than 50% of the time. Furthermore, after phase swapping, the average power for the three phases are 66.22kW, 66.64kW, and 65.04kW, respectively – the difference is below 2.5%. The remaining random imbalance requires demand-side response to address, if the three phases are to be fully rebalanced.

Based on the above results, it is concluded that the degree of power imbalance provides a useful guidance for phase swapping – this validates the methodology proposed by this paper.

H. Discussions

Among the 782 substations, 235 (30.1%), 164 (21.0%), 170 (21.7%), and 213 (27.2%) of them belong to the definite-max, definite-order, definite-min, and random imbalance scenarios, respectively. This indicates that the majority (72.8%) of the low voltage substations have SIB that can be addressed by phase swapping.

Three-phase power imbalance is directly associated with the costs for distribution network operators, including energy losses along the neutral wire and additional network investment costs. Although there is no regulatory limit on power imbalance, it will save costs for distribution network operators to rebalance three-phase power. A key value of this work is therefore to guide phase rebalancing practices. Furthermore, a few references [13], [15], [23] also focus on three-phase power imbalance.

It is necessary to judge the scenario according to the a priori judgment criteria before performing the decomposition. The necessity of the a priori judgment is because of the nature of three-phase power imbalance, i.e., the fact that any set of three-phase power series belong to one and only one of the four mutually exclusive scenarios. Once the scenario is determined, the three-phase power decomposition is definite as the closed-

form solution to the quadratic optimization problem of the scenario.

Phase swapping (also known as rephasing) is a popular technique to rebalance three-phase supply in the medium-to-long term [9], [11], [25]. It requires scheduled outage, the time of which can be carefully chosen to minimize the impact on customers.

The degree of power imbalance based on the SIB component reveals the maximum potential (also the maximum need) of phase swapping. However, it does not mean that phase swapping will always meet the maximum need in practice. Rather, it is common for phase swapping to mitigate the SIB but not completely eliminate it – in this case demand-side managements will be required to resolve the residual SIB and the RIB, if the three phases are to be fully rebalanced; alternatively, phase swapping may deliver more than the maximum need by swapping too much, causing an overcompensation that requires demand-side managements to further rebalance the three phases.

The RIB component reveals the minimum need for demand-side managements. If phase swapping exactly meets the maximum need as indicated by the degree of power imbalance, then demand-side managements only need to reduce the loads equal to the RIB component for each phase. However, because of the imperfect phase swapping in practice (as explained above), the actual need for demand-side managements is likely to be greater than the RIB component.

It should be noted that reactive power also affects network loading and phase imbalance. A major obstacle to quantifying the time-varying power factor (hence the time-varying reactive power) in real-time operation is the lack of phasor measurements in distribution networks, especially in low voltage networks. Therefore, there is hardly any information on power factor (reactive power) in real-time operation. A feasible solution is to assume an average power factor: existing publications [9], [25], [11] on phase rebalancing considers load patterns represented by active power only, based on an implicit assumption of an average power factor. Reference [1] also assumes an average power factor when calculating loading levels. Reference [26] derived an average power factor of 0.9 for residential customers. Assuming such an average power factor, if the active power is rebalanced, the reactive power is automatically rebalanced. Hence, the decomposition method needs to consider three-phase active power only.

The three-phase power decomposition method proposed by this paper is not limited to substations, but is equally applicable to nodes along the feeder with three-phase power measurements. A major obstacle to understanding the phase imbalance along feeders beyond substations is the lack of monitoring along low voltage (415V) feeders. Only a selection of the UK's low voltage substations are monitored [27], because of cost barriers. Furthermore, existing publications [13], [15] focus on phase rebalancing at the substation side to prevent the imbalance from propagating to higher-level networks. Otherwise, three-phase power imbalance will further cause energy losses and increased investment costs in higher-level networks. This research therefore focuses on three-phase

power imbalance at the substation side, using the available substation-side data provided by Western Power Distribution.

VIII. CONCLUSIONS

This paper identifies the systematic imbalance component and random imbalance component from any set of three-phase power series. The systematic component, as a direct consequence of uneven load allocations, can be addressed by phase swapping; the random imbalance component, as a result of random individual load fluctuations, requires demand-side managements, if the three phases are to be fully rebalanced. A new a priori judgment method is proposed to classify any set of three-phase power series into one of the four scenarios, i.e. definite-max, definite-order, definite-min, and random imbalance scenarios, by judging both the percentage of time and the average power to ensure robustness. For each scenario except the random imbalance one, a novel decomposition method is proposed to decompose three-phase power series into a systematic imbalance component and a random imbalance component, which are the closed-form solution to a quadratic optimization problem that minimizes the random imbalance component. The degree of power imbalance is defined for each scenario based on the systematic imbalance component.

Case studies demonstrate that 30.1%, 21.0%, 21.7%, and 27.2% of 782 low voltage substations belong to the definite-max, definite-order, definite-min, and random imbalance scenarios, respectively. Decompositions are applied to the first three groups and the degree of power imbalance values are calculated based on the systematic imbalance component. The effectiveness of the degree of power imbalance as a guidance for phase swapping is validated by preliminary phase swapping.

The methodology is highly suitable for monitored low voltage distribution networks in the UK and the rest of Europe and monitored medium voltage distribution networks in the US. Distribution network operators can use the results to find out the maximum potential of phase swapping to address systematic imbalance and the minimum need for demand-side managements to address random imbalance, if the three phases are to be fully rebalanced. In addition, the degree of power imbalance not only reveals the underlying trend of systematic imbalance over time but also provides a guidance for phase swapping practices.

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BIOGRAPHIES

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